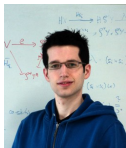


Dynamic Reductions for Model Checking Concurrent Software

Alfons Laarman
alfons@laarman.com



Henning Günther, Ana Sokolova and Georg Weissenbacher



Formal Methods in Systems Engineering
Vienna University of Technology



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Reductions

Model Checking of Concurrent Software

- Explosion of interleavings
- Partial-order reduction vs Lipton reduction
- Symbolic is a challenge
- *Global* commutativity is needed, but a severe bottleneck

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a = 1;  
b = 2;
```

||

```
x = 1;  
x += 2;  
x += 3;
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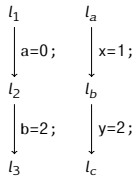
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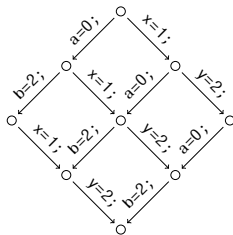
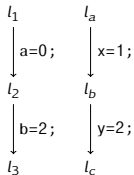
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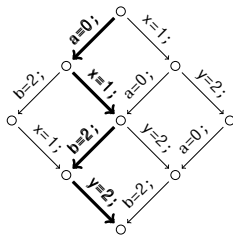
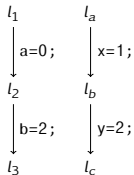
Lipton vs Partial-Order Reduction (POR)



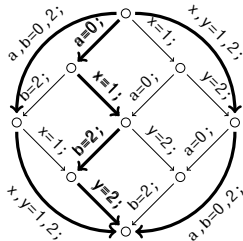
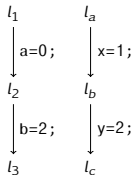
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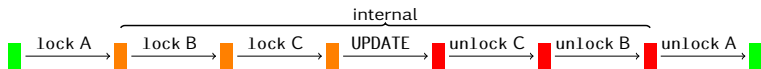
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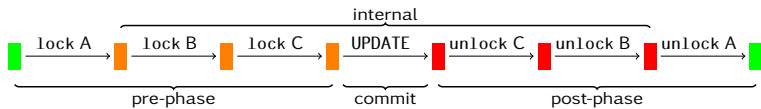
Transactions in databases



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Commutativity

Action α right commutes with β , iff α can always be delayed after β :

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Definition (Right commutativity ($\overset{\rightarrow}{\bowtie}$))

$$\alpha \overset{\rightarrow}{\bowtie} \beta \quad \text{iff} \quad \begin{array}{ccc} \sigma_1 & & \sigma_1 \xrightarrow{\beta} \sigma_4 \\ \forall \sigma_1, \sigma_2, \sigma_3 : \downarrow \bowtie & \Rightarrow \exists \sigma_4 : \downarrow \bowtie & \downarrow \bowtie \\ \sigma_2 \xrightarrow{\beta} \sigma_3 & & \sigma_2 \xrightarrow{\beta} \sigma_3 \end{array}$$

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$$\sigma_1 \quad \sigma_1 \xrightarrow{\beta} \sigma_4$$

$$\sigma_2 \xrightarrow{\beta} \sigma_3 \quad \sigma_2 \xrightarrow{\beta} \sigma_3$$

Example

- An action both-commutes with all actions that access a disjoint set of variables.
- A lock(/unlock) right(/right)-commutes with other lock and unlock operations.

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Definition (Right-Movability)

The action α of thread i is a right-mover, iff for all $j \neq i$: $\xrightarrow{\alpha} \overset{\rightarrow}{\bowtie} \rightarrow_j$

Lipton Reduction

[LIPTON '77, LAMPORT ET AL. '89]

Example (A statement sequence, where x is the only global variable)

```
a = 1; x = 2; b = 3; c = 4;
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Example (A statement sequence, where x is the only global variable)

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[LIPTON '77, LAMPORT ET AL. '89]

Lipton Reduction

A statement $\alpha_1; \dots; \alpha_n$ of thread i can be reduced to $\alpha_1 \circ \dots \circ \alpha_n$, if for some $1 \leq k < n$, and all $j \neq i$:

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$\xrightarrow{x=6} \xrightarrow{a=1} \xrightarrow{x=7} \xrightarrow{x=8} \xrightarrow{x=2} \xrightarrow{b=3} \xrightarrow{x=9} \xrightarrow{c=4}$

Lipton Reduction

[LIPTON '77, LAMPORT ET AL. '89]

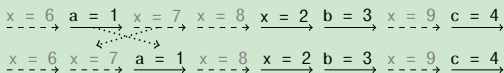
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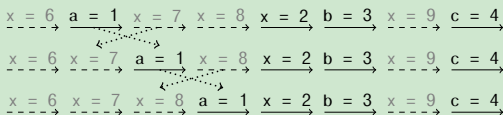
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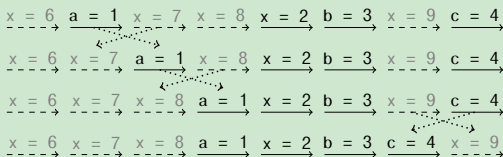
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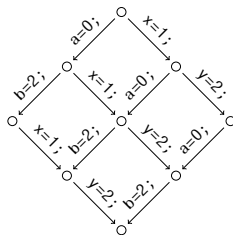
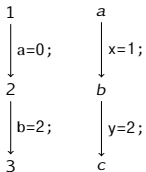
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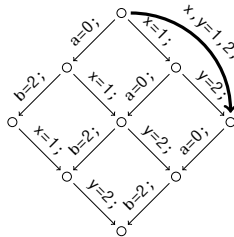
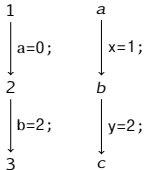
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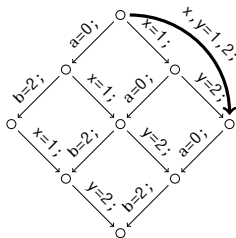
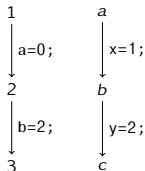
Movability is too strong a condition



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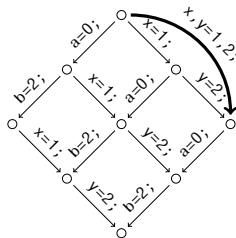
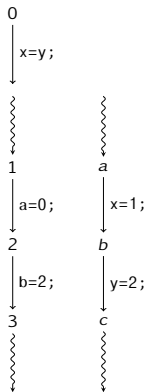


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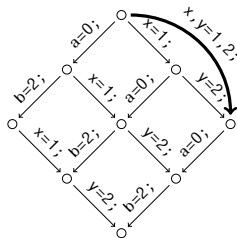
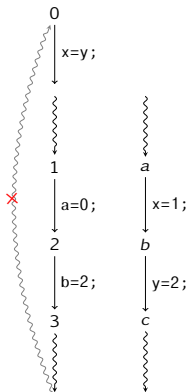
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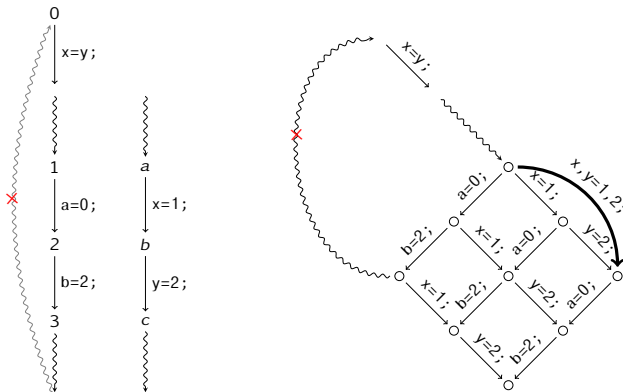
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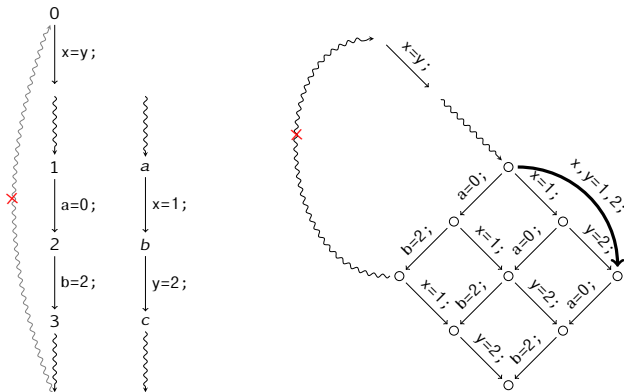
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- $\alpha_1, \dots, \alpha_{k-1} \xrightarrow{\Delta} j$ (pre-phase statements (before α_k) are *right movers*)
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Monotonicity is key!

Example

```
int *data = NULL;
void worker_thread(int tid) {
    if (data == NULL) {
        int *tmp = read_from_disk(1024);
W:    if (!CAS(&data, NULL, tmp)) free(tmp);
    }
    for (int i = 0; i < 512; i++)
R:    process(data[i + tid * 512]);
}
int main () {
    pthread_create(worker_thread, 0); // T1
    pthread_create(worker_thread, 1); // T2
}
```

Example

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int *data = NULL;
void worker_thread(int tid) {
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while (!if (!CAS(&data, NULL, tmp)) free(tmp);
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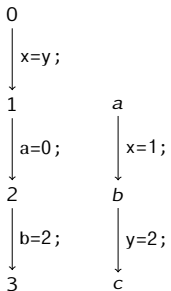
Example

Other dynamic conditions

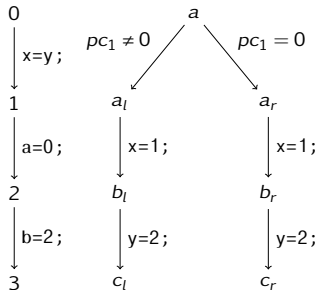
- Pointers / array indices that don't change value.
- Atomic Compare-And-Swap (CAS) operations to permanently grab resources.

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```

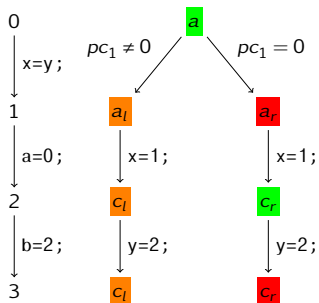
Instrumentation with dynamic reduction



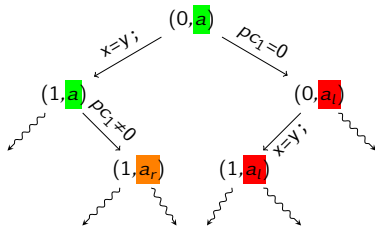
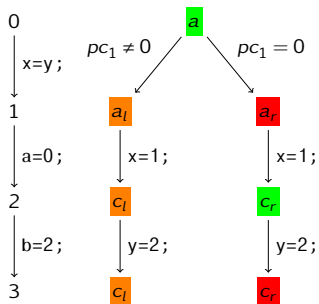
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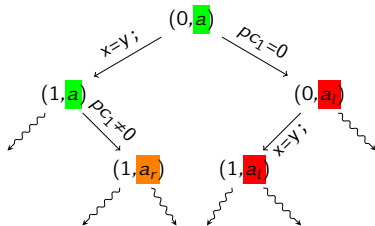
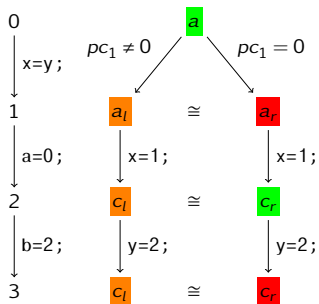
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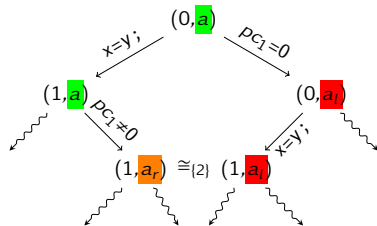
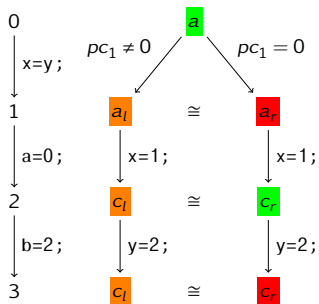
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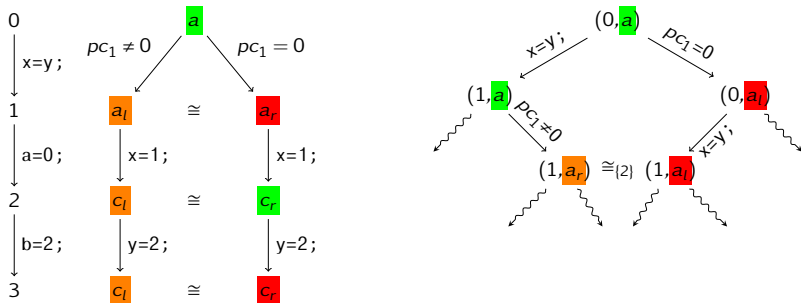
Instrumentation with dynamic reduction



Instrumentation with dynamic reduction



Instrumentation with dynamic reduction



Definition (Dynamic *both*-moving conditions)

A state predicate c_α is a dynamic both-moving condition for an action α , if for all $j \neq i$:

- $(c_\alpha // \xrightarrow{\alpha}_i) \bowtie (c_\alpha // \rightarrow_j)$
- c_α is never disabled

Instrumentation with dynamic reduction



Example (Heuristics for other dynamic conditions)

```
int T[10] = {E,E,22,35,46,25,E,E,91,E};
```

```
int find-or-put(int v) {
    int hash = v / 10;
    for (int i = 0; i < 10; i++) {
        int index = (i + hash) % 10;
        if (CAS(&T[index], E, v)) {
            return INSERTED;
        } else if (T[index] == v)
            return FOUND;
    }
    return TABLE_FULL;
}

int main() {
    pthread_create(find-or-put, 25);
    pthread_create(find-or-put, 42);
}
```

Instrumentation with dynamic reduction



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```

$c_\alpha := T[\text{index}] \neq E$

Movability up to bisimulation

Definition (Right-commutativity up to bisimulation ($\overset{\rightarrow}{\bowtie}_X$))

The transition relation $\overset{\alpha}{\rightarrow}_i$ right-commutes with $\overset{\beta}{\rightarrow}_j$ up to \cong_X ,

notation $\overset{\alpha}{\rightarrow}_i \overset{\rightarrow}{\bowtie}_X \overset{\beta}{\rightarrow}_j$, iff:

$$\forall \sigma_1, \sigma_2, \sigma_3: \begin{array}{c} \sigma_1 \\ \downarrow \alpha \\ \sigma_2 \end{array} \overset{\beta}{\rightarrow}_j \sigma_3 \quad \Rightarrow \quad \exists \sigma'_3, \sigma_4: \begin{array}{c} \sigma_1 \overset{\beta}{\rightarrow}_j \sigma_4 \\ \downarrow \alpha \quad \downarrow \beta \\ \sigma_2 \overset{\beta}{\rightarrow}_j \sigma_3 \cong_X \sigma'_3 \end{array}$$

Definition (Right-movability up to bisimulation)

The action α of thread i is a right-mover up to \cong_X , iff for all $j \neq i$:

$$\overset{\alpha}{\rightarrow}_i \overset{\rightarrow}{\bowtie}_X \rightarrow_j$$

Transaction Reduction

[FLANAGAN, QADEER SOFTMC'03]

Theorem (Reduction)

Let (\rightarrow, S) be a transition system. For all $i, j \neq i$, let $S = R_i \uplus L_i \uplus N_i$ such that

- $L_i // \rightarrow_i \parallel R_i = \emptyset$ *post does not reach pre*
- $\rightarrow_i \parallel R_i \overset{\rightarrow}{\bowtie} \rightarrow_j$ *\rightarrow_i into pre right commutes with \rightarrow_j*
- $L_i // \rightarrow_i \overset{\leftarrow}{\bowtie} \rightarrow_j$ *\rightarrow_i from post left commutes with \rightarrow_j*
- $\forall \sigma \in L_i : \exists \sigma' \in N_i : \sigma \rightarrow_i^* \sigma'$ *post phases terminate*

Let $\hookrightarrow_i \triangleq \bigcup_{j \neq i} N_j // \rightarrow_i$ (*i can only transition when all j are in an external state*).

Suppose $\sigma \rightarrow^* \sigma', \sigma \in N$ and $\sigma' \in N$, then there is an $\sigma'' \in N$ s.t. $\sigma \hookrightarrow^* \sigma''$.

Dynamic Transaction Reduction

[GÜNTHER, LAARMAN, SOKOLOVA, WEISSENBACHER VMCAI'17]

Theorem (Reduction)

Let $(\rightarrow, \mathcal{S})$ be a transition system. For all $i, j \neq i$, let $\mathcal{S} = R_i \uplus L_i \uplus N_i$ such that
 For each thread i , there exists a thread bisimulation relation \cong_i , and:

- \bullet $L_i \parallel \rightarrow_i \parallel R_i = \emptyset$ *post does not reach pre*
- \bullet $\rightarrow_i \parallel R_i \overset{\rightarrow}{\bowtie} \{j\} \rightarrow_j$ *\rightarrow_i into pre right commutes with \rightarrow_j*
- \bullet $L_i \parallel \rightarrow_i \overset{\leftarrow}{\bowtie} \{i, j\} \rightarrow_j$ *\rightarrow_i from post left commutes with \rightarrow_j*
- \bullet $\forall \sigma \in L_i : \exists \sigma' \in N_i : \sigma \rightarrow_i^* \sigma'$ *post phases terminate*
- \bullet $\cong_i \subseteq L_j^2 \cup R_j^2 \cup N_j^2$ *(\cong_i entails j -phase-equality)*

Let $\hookrightarrow_i \triangleq \bigcup_{j \neq i} N_j \parallel \rightarrow_i$ (i can only transition when all j are in an external state).

Let $\rightsquigarrow_i \triangleq N_i \parallel (\hookrightarrow_i \parallel \overline{N_i})^* \hookrightarrow_i \parallel N_i$ (block steps skip internal states)

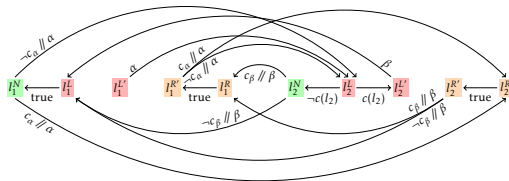
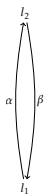
Suppose $\sigma \rightarrow_i^* \sigma', \sigma \in N$ and $\sigma' \in N$, then there is an $\sigma'' \in N$ s.t. $\sigma \rightsquigarrow_i^* \sigma''$.

Complete instrumentation

$G_i \triangleq (V_i, \delta_i)$	V'_i, δ' in G'_i (pictured)
$\forall (l_a, \alpha, l_b) \in \delta_i:$	
$\forall (l_a, \alpha, l_b) \in \delta_i:$	
$\forall l_a \in V_i:$	$l_a^R \xrightarrow{\text{true}} l_a^{R'}$
$\forall l_a \in V_i \setminus LFS_i:$	<p>with $c(l_a) \triangleq \bigwedge_{(l_a, \alpha, l_b) \in \delta_i} c_\alpha$</p>
$\forall (l_a, \alpha, l_b) \in \delta_i, l_a \in V_i \setminus LFS_i:$	$l_a^{L'} \xrightarrow{\alpha} l_b^L$
$\forall l_a \in LFS_i:$	$l_a^L \xrightarrow{\text{true}} l_a^N$

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VVT

Vienna Verification Tool

[GÜNTHER, LAARMAN, WEISSENBACHER SVCOMP'16]

<http://vvt.forsyte.at/> (open source)

VVT

Vienna Verification Tool

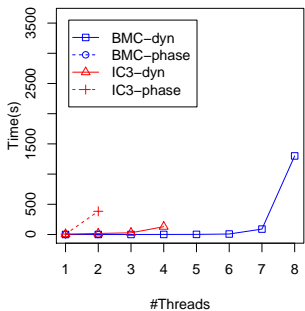
[GÜNTHER, LAARMAN, WEISSENBACHER SVCOMP'16]

<http://vvt.forsyte.at/> (open source)

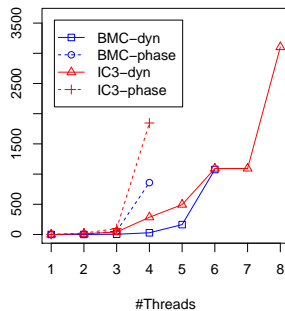
- BMC with all dynamic reductions (*BMC-dyn* in the graphs);
- BMC with only static reductions (*BMC-phase*);
- IC3 with all dynamic reductions (*IC3-dyn*); and
- IC3 with only static reductions (*IC3-phase*).

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Lazy initialization



Dynamic locking

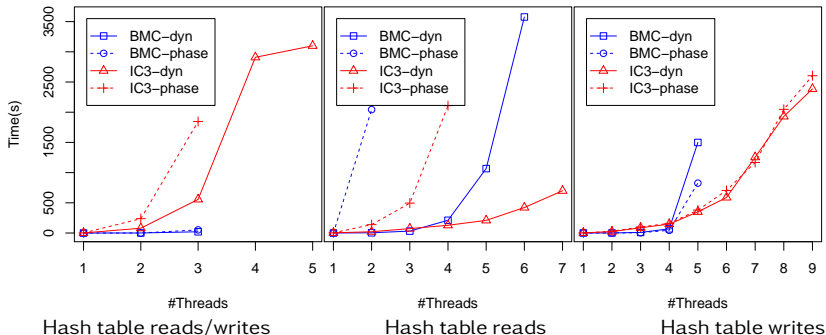
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Conclusions

Contributions

- Dynamic movers as in stubborn set POR
- “Straight-forward” instrumentation and encoding
- Dynamic mover conditions are suitable for symbolic model checking

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Open Questions

- Is a weaker reduction theorem possible?
- How do transactions compare to POR?