Multi-Core Partial-Order Reduction for LTL Model Checking

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joint work with **Anton Wijs** (Eindhoven University of Technology)

FAKULTÄT FÜR !NFORMATIK

Formal Methods in Systems Engineering
Vienna University of Technology

for(syte Formal Methods in Systems Engineering

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Setting: Explicit-State Model Checking

State-space graph: $\mathcal{G} = (\mathcal{S}, T, s_0)$
On-the-fly exploration: $en : \mathcal{S} \to \mathcal{S}$

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For LTL, a Büchi automaton with $\mathcal{F} \subseteq \mathcal{S}$
[Vardi, 1996 – An automata-theoretic approach to LTL]

Scalable Multi-Core Model Checking

Research questions

- Can model checking scale (linearly, ideally) on modern multi-cores?
- Are our parallel solutions compatible with different optimizations?

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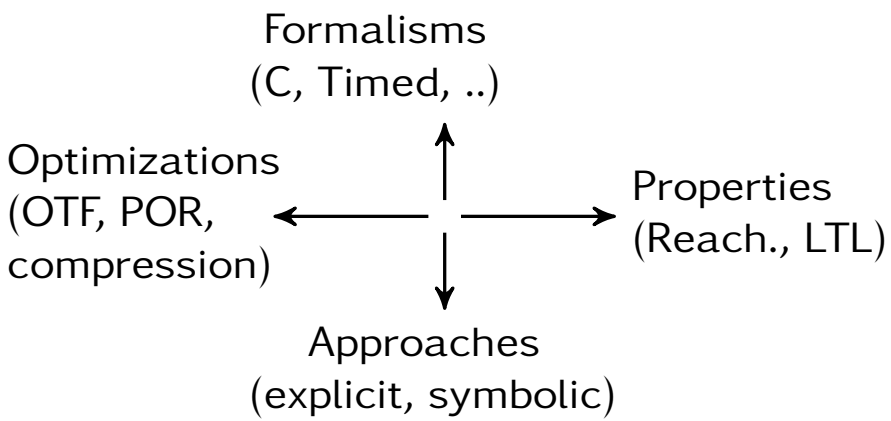

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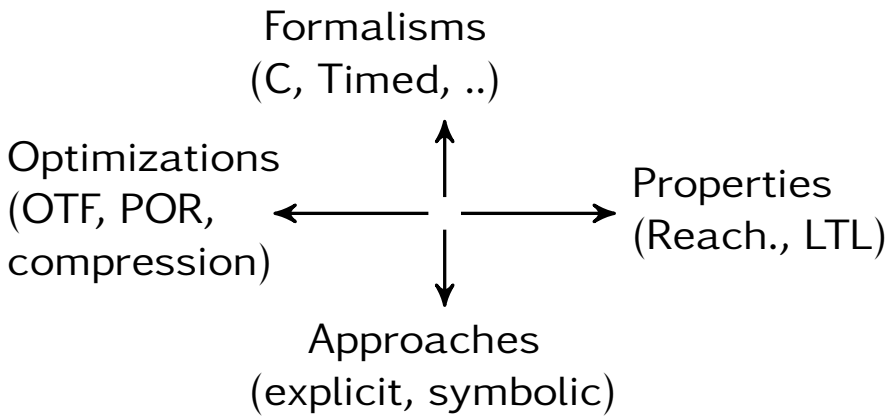

Formalism	Property	Explicit state	+ Compression	+ On-the-fly	+ POR	Symbolic
Plain	Reachability	1/2	?	?	1/2	?
	Liveness	1/2	?	?	?	?
Timed	Reachability	?	?	?	?	?
	Liveness	?	?	?	?	?

Status 2009:
[Holzmann,Bošnački, 2007 – Multi-Core Model Checking with SPIN]
[Barnat et al., 2010 – Scalable Shared Memory LTL Model Checking]

PhD Project

1. Shared hash table approach (as opposed to distributed algorithms)
2. Lockless data structures
3. Parallel algorithms

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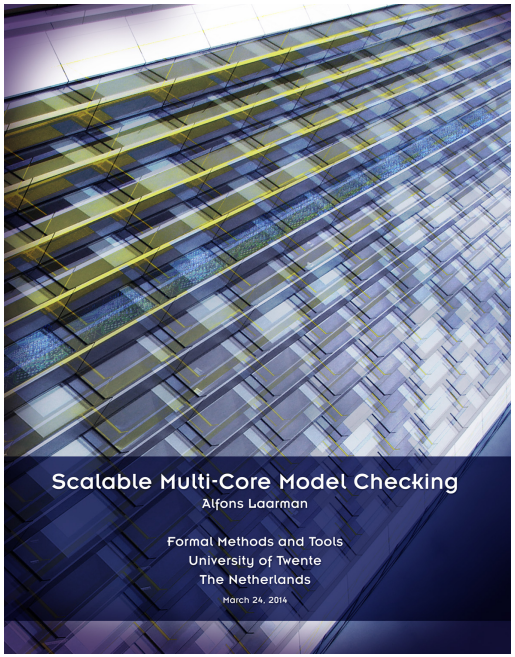
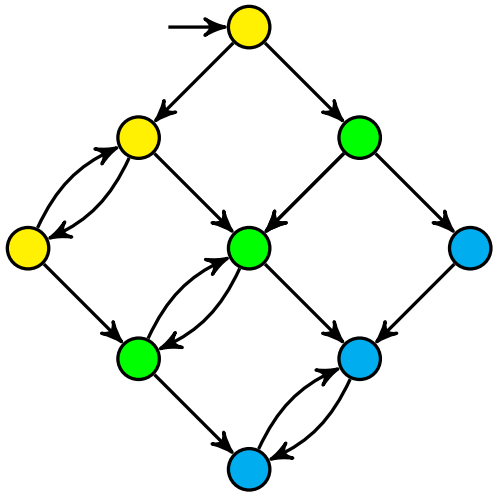


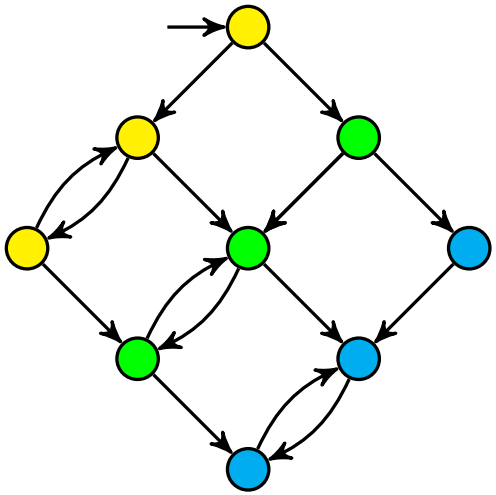
[Laarman, 2014 – Scalable Multi-Core Model Checking]
[vDijk, Laarman, vdPol, 2012 – Multi-Core BDD Operations for Symbolic Reachability]

Partial-Order Reduction for LTL

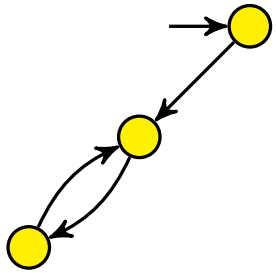


Partial-Order Reduction for LTL

Reduce successor function: $por(s) \subseteq en(s)$.



deadlock
$\longrightarrow$



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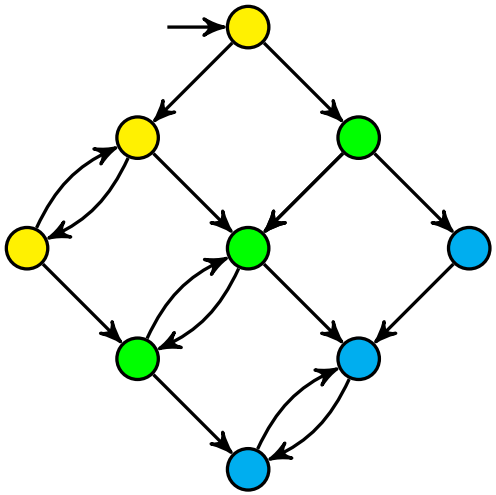

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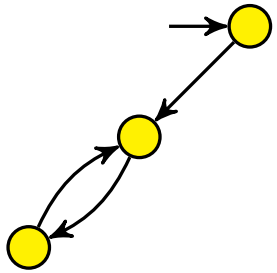
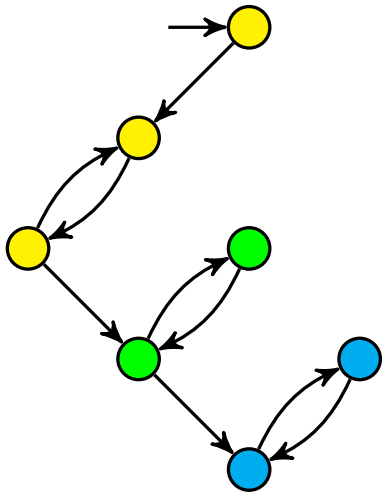


$\downarrow$ +ignoring



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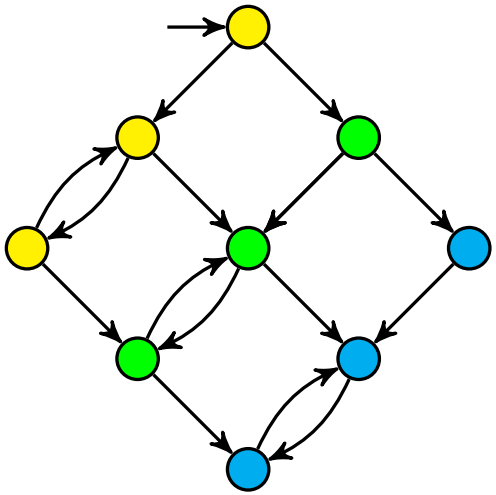

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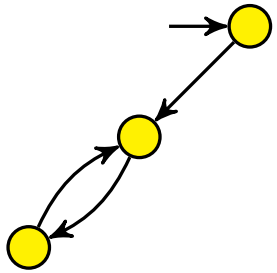

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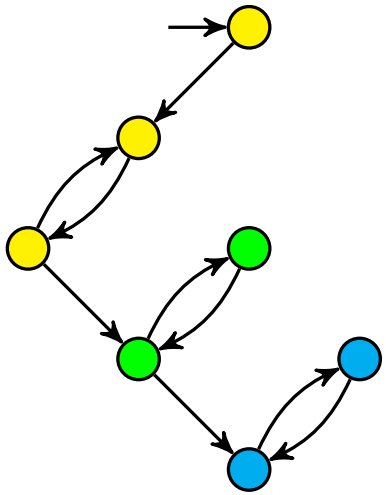

Smaller reduced set $por()$ leads to smaller state space.

DFS Stack Proviso

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procedure DFS(s)
    for all s′ in por(s) do
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    if successor of s is on the stack then
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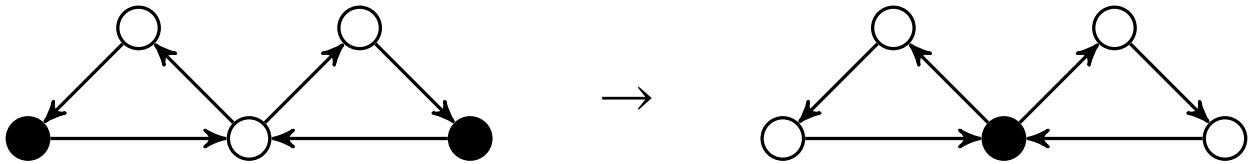

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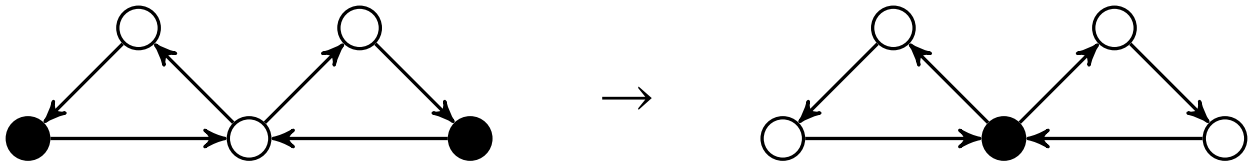

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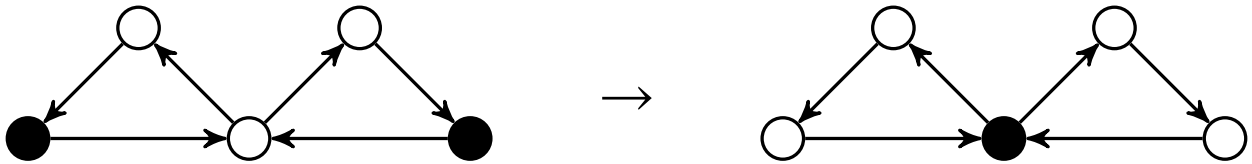

Problem

- (Minimal) feedback vertex set (FVS) → NP-complete
- Stack proviso is the best we can do on-the-fly
- DFS is P-complete ⇒ inherently sequential

Related Work

Algorithm/Proviso	Reduction	Scalability
NDFS/Stack	++	n/a
?? / TwoPhase [Gopalakrishnan et al.]	+-	??
OWCTY/Topological sort [Barnat et al.]	+-	+

Related Work

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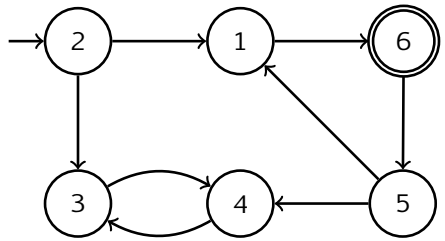
OWCTY/Topological sort [Barnat et al.]	+-	+
MC-NDFS/n/a	n/a	++

Challenge: do as good as DFS stack proviso in the parallel setting

Nested Depth-First Search for LTL

[Courcoubetis'93]

Accepting cycle detection in Büchi automaton:



Nested Depth-First Search for LTL

[Courcoubetis'93]

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procedure DFSblue(s)
    s.cyan := true
    for all s′ in en(s) do
        if ¬s′.blue∧¬s′.cyan then
            DFSblue(s′)
    if s ∈ F then
        DFSred(s)
    s.blue := true
    s.cyan := false
procedure DFSred(s)
    s.red := true
    for all s′ ∈ en(s) do
        if s′.cyan then ExitCycle
        if ¬s′.red then DFSred(s′)
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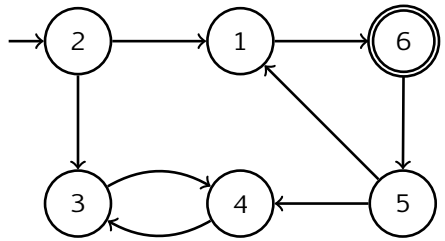

Nested DFS (NDFS)

- Linear time

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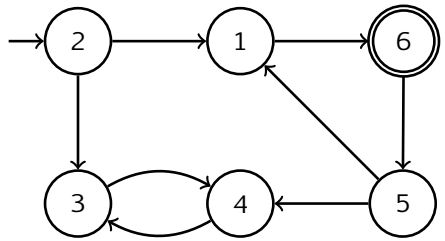

Nested DFS (NDFS)

- Linear time
- DFS itself is likely not parallelizable
  - DFS order is P-complete
  - We assume: P ≠ NC

Swarm Nested Depth-First Search

[Holzmann, 2010]

code for worker p:

```
procedure DFSblue(s, p)
    s.cyan[p] := true
    for all s′ in shuffle(en(s)) do
        if ¬s′.blue[p]∧¬t.cyan[p] then
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    if s ∈ F then
        DFSred(s, p)
    s.blue[p] := true
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procedure DFSred(s, p)
    s.red[p] := true
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Multi-core Nested Depth-First Search

[ATVA11], [PDMC11], [ATVA12]

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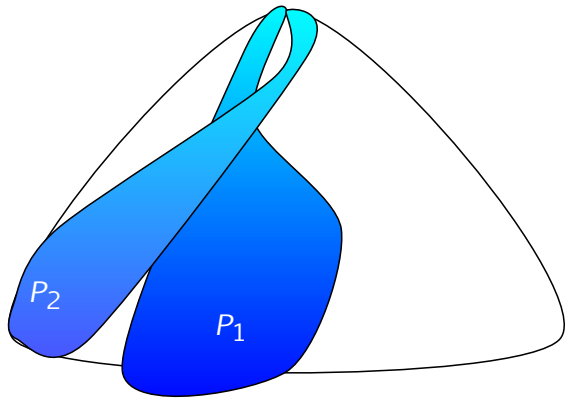

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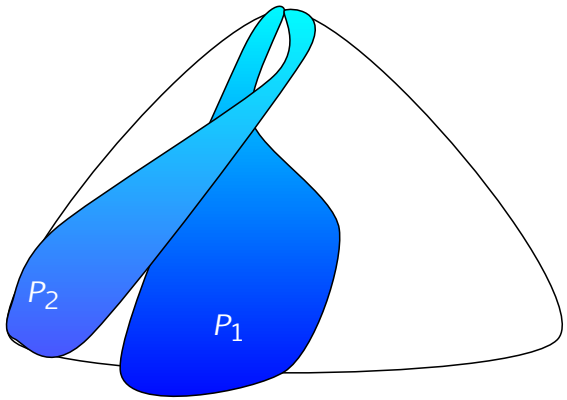

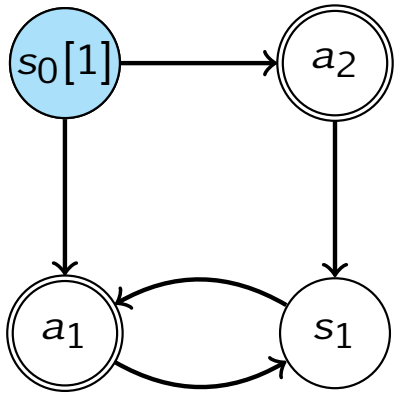

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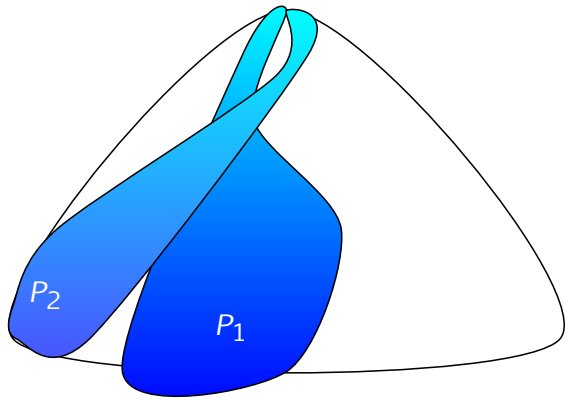

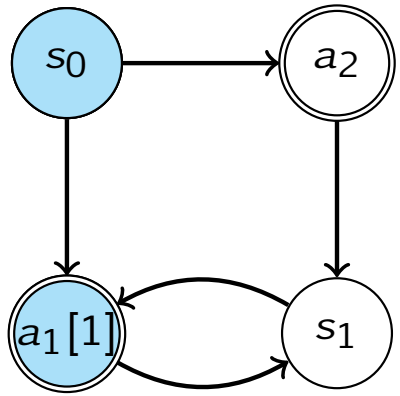

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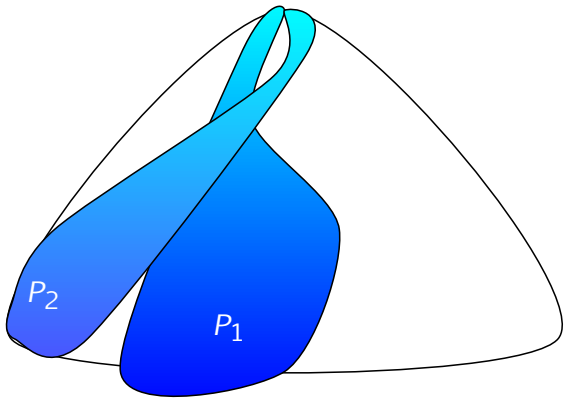

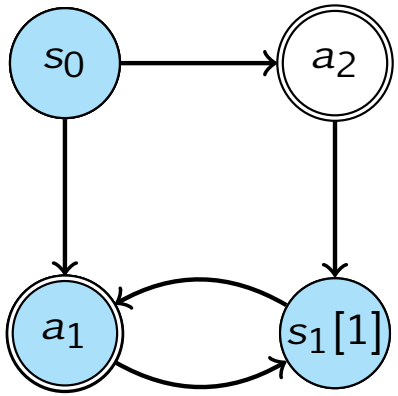

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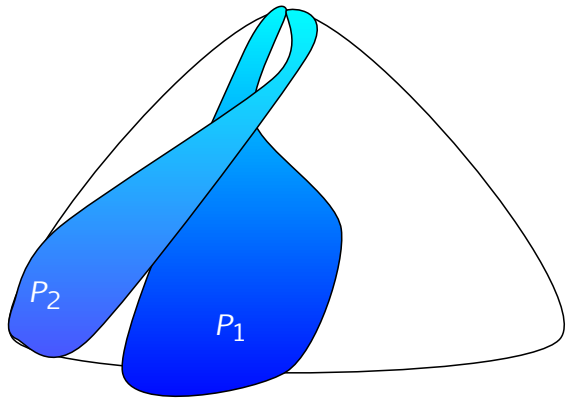

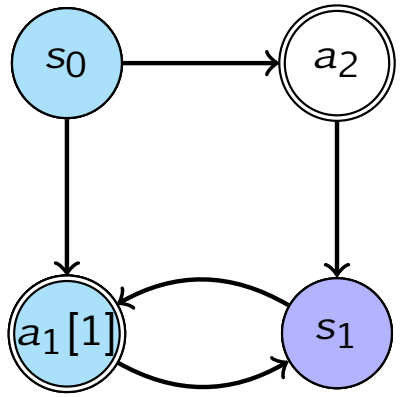

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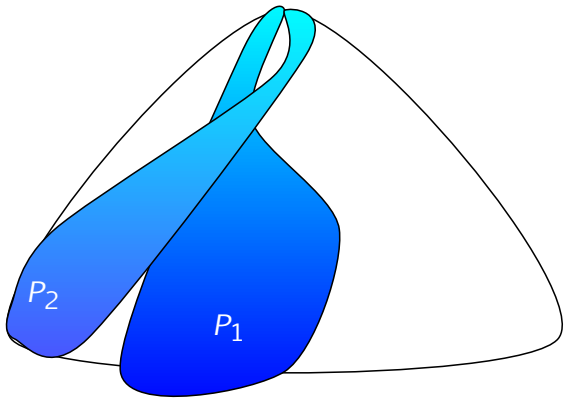

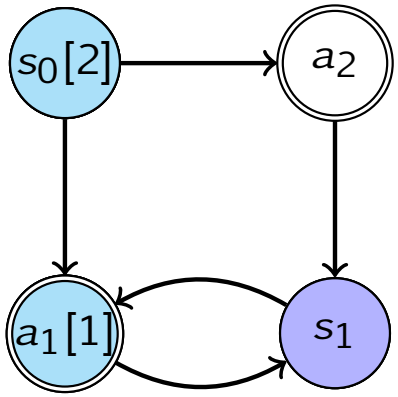

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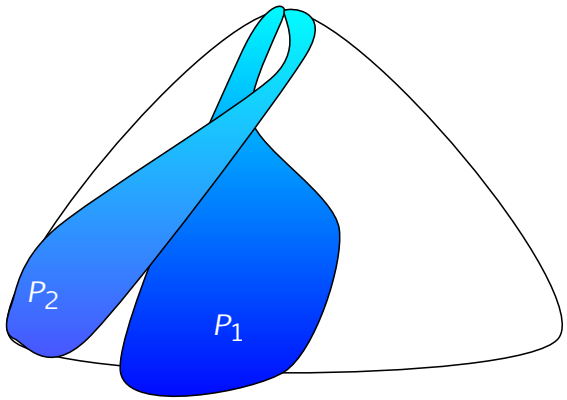
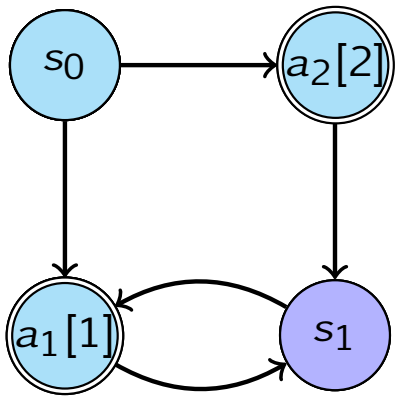
procedure DFSred(s, p)
  $R := R \cup \{s\}$
  for all s′ ∈ shuffle(en(s)) **do**
    if s′.cyan[p] **then** ExitCycle
    if $s' \notin R \wedge \neg s.red$ **then** DFSred(s′, p)





Multi-core Nested Depth-First Search

[ATVA11], [PDMC11], [ATVA12]

code for worker p:

procedure DFSblue(s, p)
  s.cyan[p] := true
  for all s′ **in** shuffle(en(s)) **do**
    if ¬s′.blue∧¬t.cyan[p] **then**
      DFSblue(s′, p)
  if $s \in \mathcal{F}$ **then**
    $R := \emptyset$
    DFSred(s, p)
    await all accepting in $R \setminus \{s\}$ **are red**
    mark all in R red
  s.blue := true
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procedure DFSred(s, p)
  $R := R \cup \{s\}$
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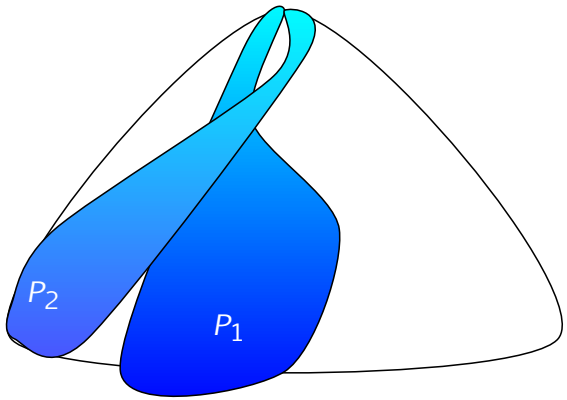

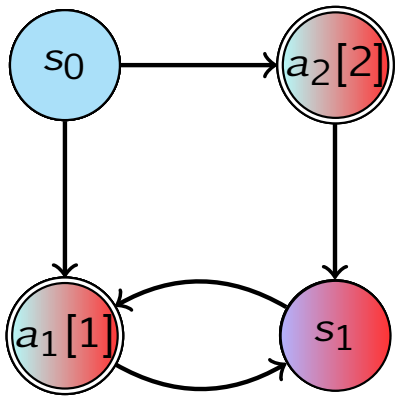

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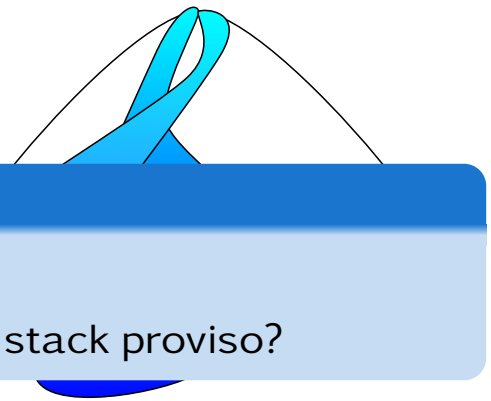
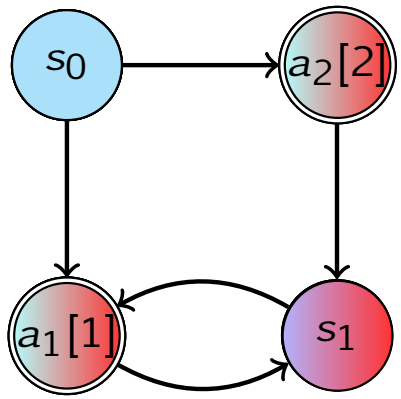


Conclusions

- MC-NDFS scales in practice and uses DFS
- Does it preserve enough order to implement stack proviso?



Stack Proviso in Parallel

```
procedure DFSblue(s, p)
    s.cyan[p] := true
    for all s′ in shuffle(por(s)) do
        if ¬s′.blue∧¬t.cyan[p] then
            DFSblue(s′, p)
    if s ∈ F then
        R := ∅
        DFSred(s, p)
        await all accepting in R \ {s} are red
        mark all in R red
    if ∃s′ ∈por(s): s′.cyan then
        explore s fully with DFSblue
    s.blue := true
    s.cyan[p] := false
```

Stack Proviso in Parallel

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procedure DFSblue(s, p)
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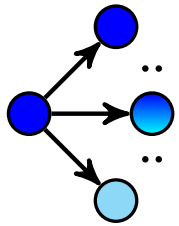
    s.blue := true
    s.cyan[p] := false
```

Completeness

$Blue \subseteq \Box(Blue \cup \bigcup_p Cyan_p)$



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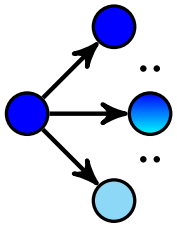

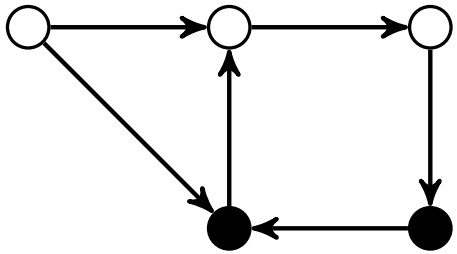

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        DFSred(s, p)
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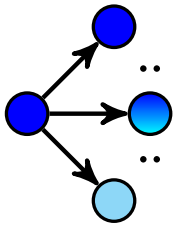
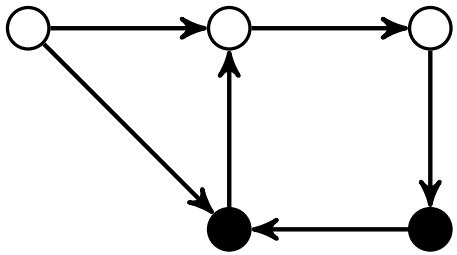
    R := R ∪ {s}
    for all s′ ∈ shuffle(por(s)) do
        if s′.cyan[p] then ExitCycle
        if s′ ∉ R ∧ ¬s.red thenDFSred(s′, p)
    if successor of s is on DFSred_p stack then
        explore s fully with DFSred
```

Completeness

$Blue \subseteq \Box(Blue \cup \bigcup_p Cyan_p)$





Re-visiting problem
[Holzmann et al., 1996 –
On nested depth-first search]

Stack Proviso in Parallel

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        if s′ ∉ R ∧ ¬s.red thenDFSred(s′, p)
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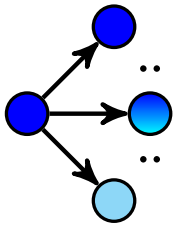

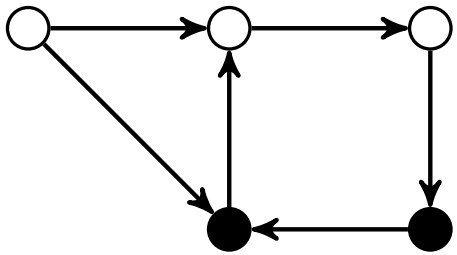

Re-visiting problem
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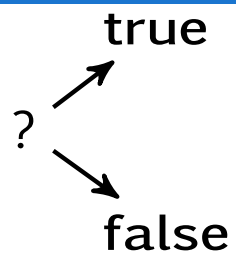
No termination!

The Parallel Cycle Proviso

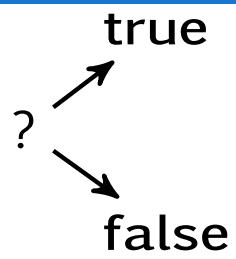
Add a state proviso flag:



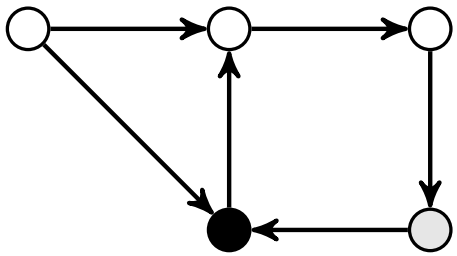

```
procedure dfsBlue(s, p)
    ...
    prov := successor of s is on the local DFSblue stack
    compare_and_swap (s.proviso, ?, prov)
    if s.proviso then
        explore s fully with dfsRed
    ...
procedure dfsRed(s, p)
    ...
    prov := successor of s is on the local DFSred stack
    compare_and_swap (s.proviso, ?, prov)
    if s.proviso then
        explore s fully with dfsRed
    ...
```

The Parallel Cycle Proviso

Add a state proviso flag:



Performance



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The Parallel Cycle Proviso

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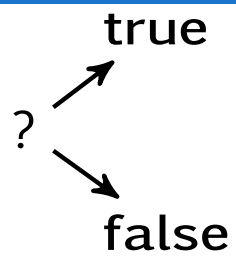


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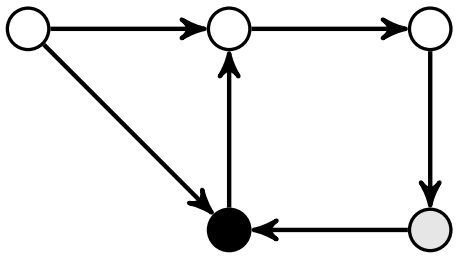

Correctness

Backtracked states (blue and red):
$F = \{s \mid s.proviso \neq ?\}$
$V = \{s \mid s.proviso = \textbf{false}\}$

Lemma 6. Backtracked states have a proviso set: $(B \cup R) \subseteq F$.

Lemma 8. Successors of V states are backtracked: $V \subseteq \Box(B \cup R)$.

Conclusions

Parallel cycle proviso (% reduction)

Model	Threads 1	64
leader_filters.7	2.35	2.35
elevator.3	94.20	94.96
leader_election.4	3.02	3.02
leader_election.6	0.70	0.69
anderson.6	48.43	51.71
garp	18.69	20.79
peterson4	15.52	15.67
iprotocol-2	34.80	37.91
pacemaker_distributed	47.81	48.26
pacemaker_concurrent	45.90	46.00

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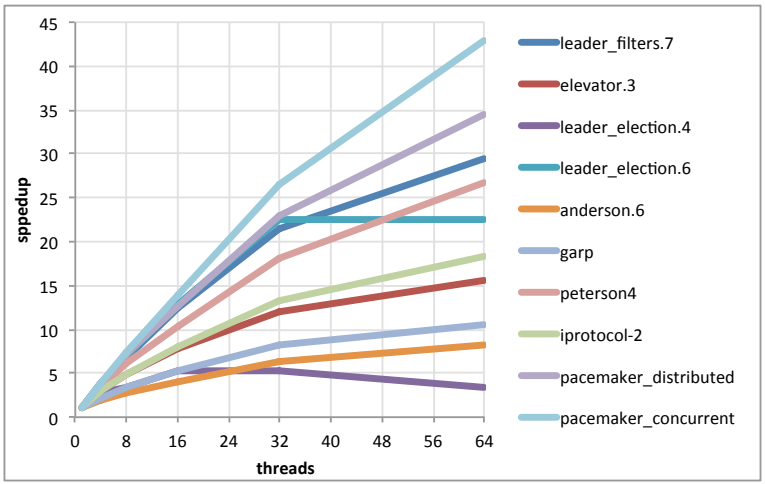

- DFS-proviso's reduction power is preserved
- Speedups maintained

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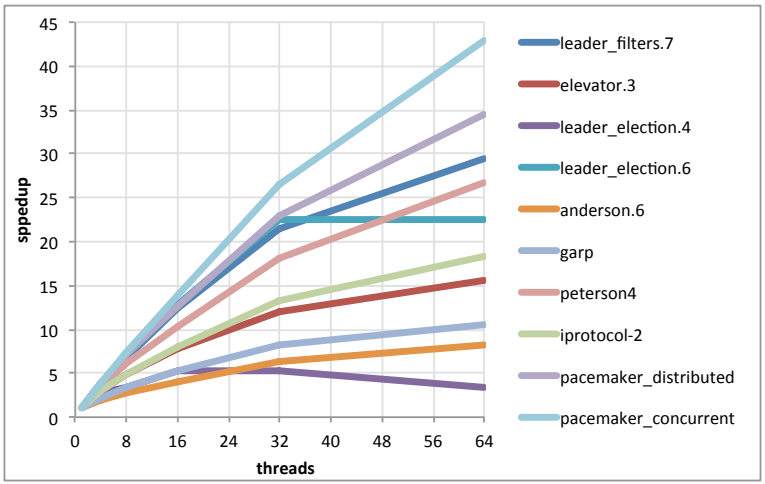

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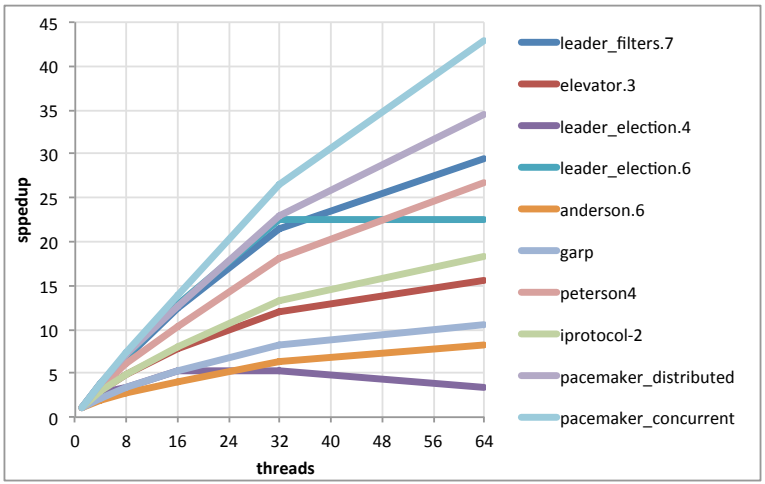

- DFS-proviso's reduction power is preserved
- Speedups maintained
- Demonstrates the strength of parallel DFS-based algortihms
- How much of the DFS order is preserved?
- On which type of graphs does CNDFS scale?