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formal methods & tools.

Improved Multi-core Nested Depth-First Search

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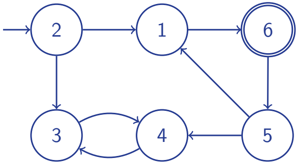
Oct 5, 2012

ATVA'12, Thiruvananthapuram, India

LTL Model Checking

- A buggy run in a system can be viewed as an infinite word
- Absence of bugs: emptiness of some Büchi automaton [Vardi/Wolper 86]
- Graph problem: find a reachable accepting state on a cycle
- Basic algorithm: Nested Depth-First Search (NDFS)



LTL Model Checking is Finding Accepting Cycles

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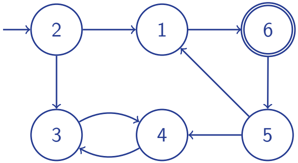

This talk

- Traditional solution: NDFS
- Multi-core NDFS flavors
- CNDFS and a comparison

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    for all t ∈ post(s) do
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- Linear time, on-the-fly
- Blue is inherently depth-first

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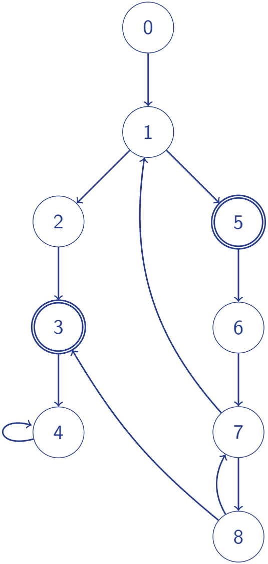

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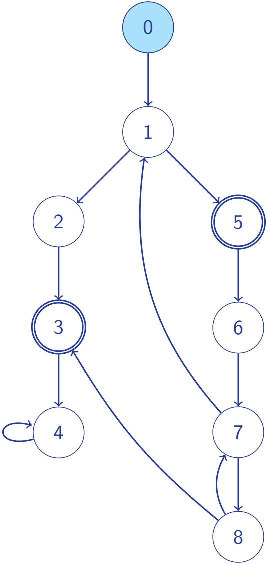

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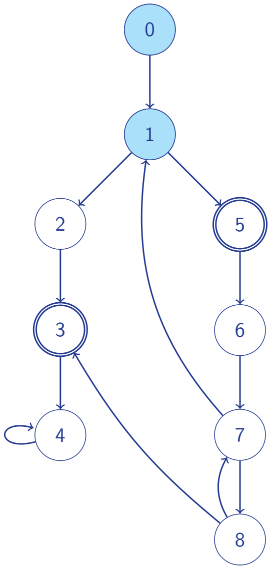

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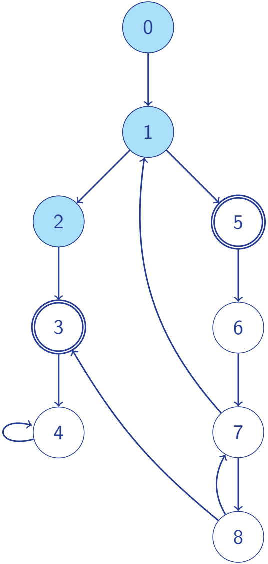

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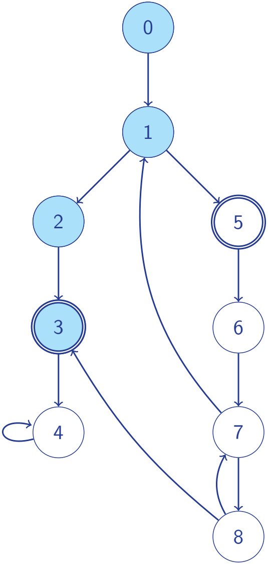

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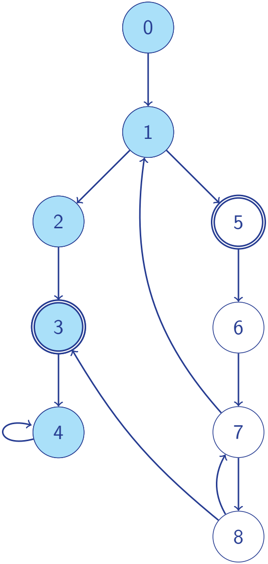

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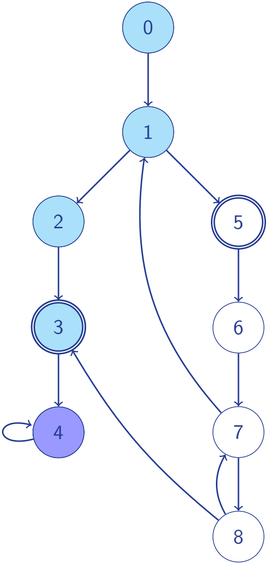

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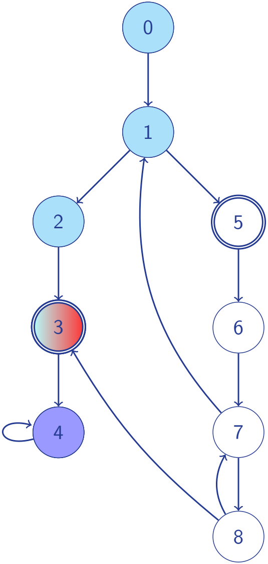

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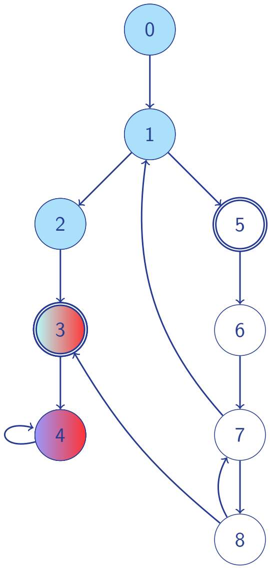

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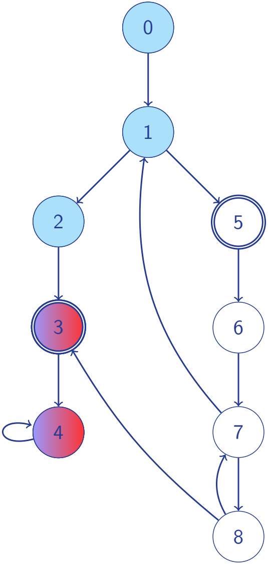

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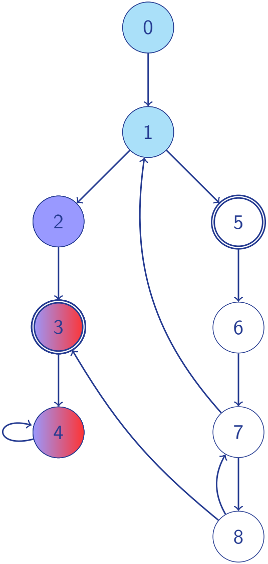

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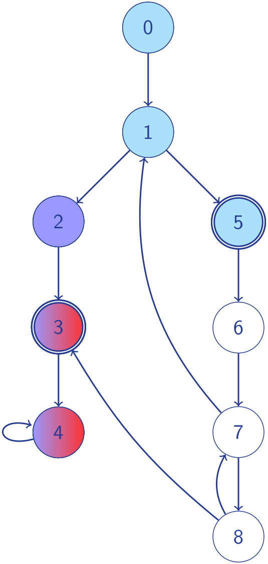

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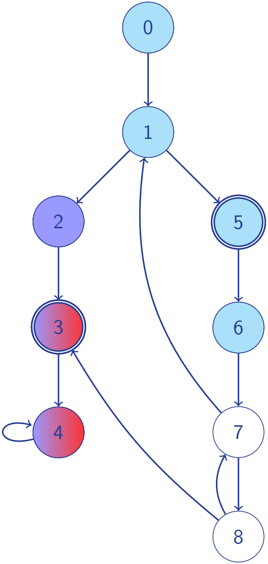

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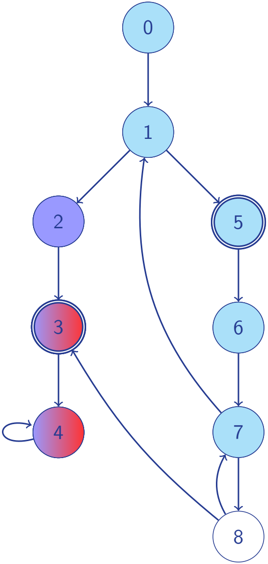

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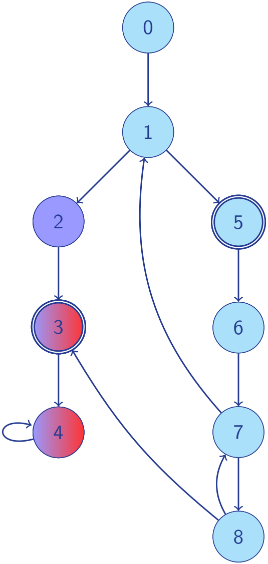

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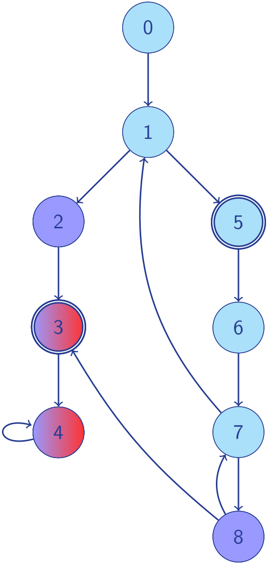

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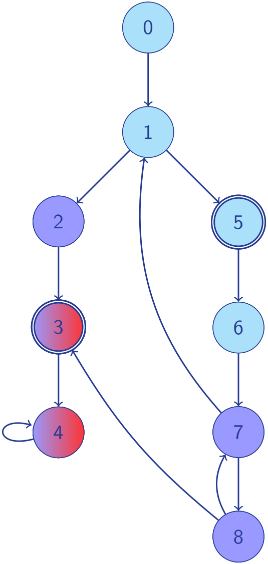

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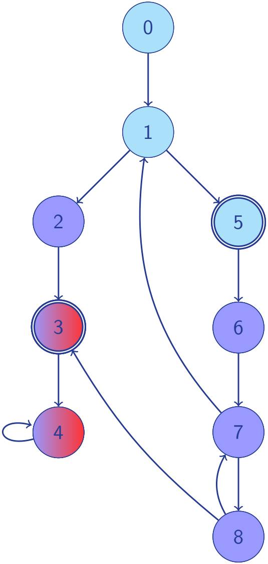

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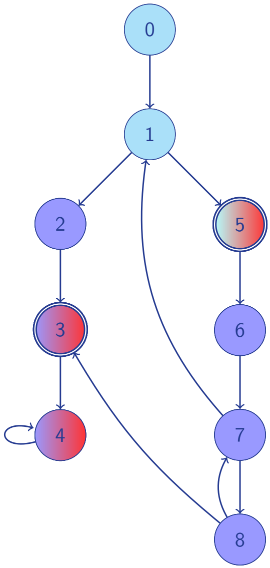

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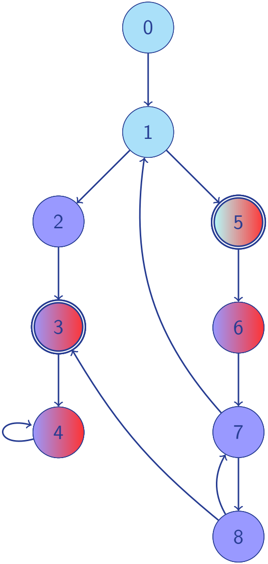

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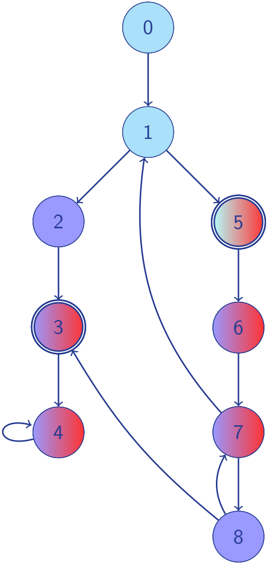

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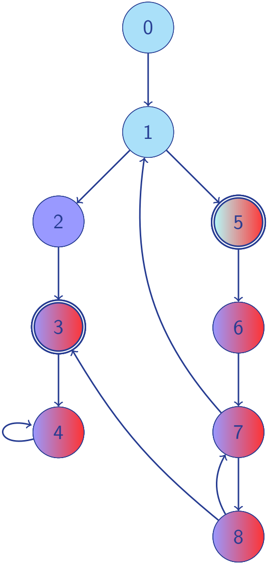

NDFS example

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- Memory usage: 2 bits per state [Schwoon et al.]
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- Memory usage: 2 bits per state [Schwoon et al.]
- DFS-order $\implies$ Hard to parallelize
- Completeness: DFS-order $\implies$ one accepting state in each red search (safe to reuse the red color)

Swarmed and Multi-core Nested Depth-First Search

code for worker i:

```
procedure DFSblue(s,i)
    s.cyan[i] := true
    for all t in shuffle(post(s)) do
        if ¬t.blue[i] ∧ ¬t.cyan[i] then
            DFSblue(t,i)
    if s ∈ Accepting then
        DFSred(s,i)
    s.blue[i] := true
    s.cyan[i] := false
procedure DFSred(s,i)
    s.red[i] := true
    for all t ∈ post(s) do
        if t.cyan[i] then ExitCycle
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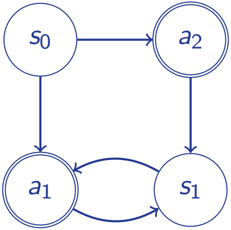
Multi-core NDFS

- Independent (swarmed) $\Longrightarrow$ no speedup for full verification
- States in shared hash table [FMCAD10]
- Share red $\Longrightarrow$ good speedup iff $|Red| = |Blue|$
- Share blue $\Longrightarrow$ lose post-order

Sharing Blue Counter Example



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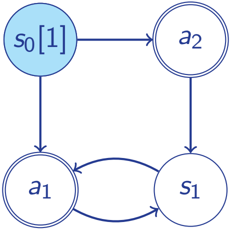

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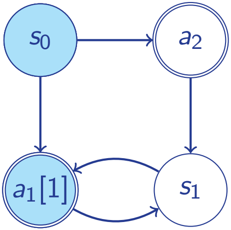

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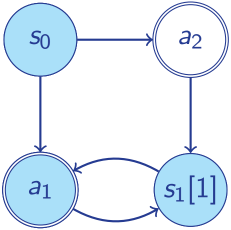

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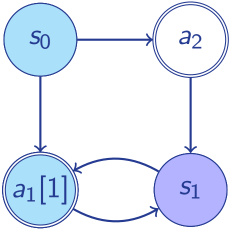

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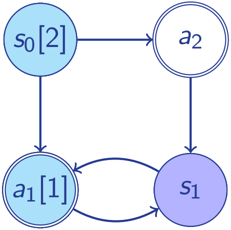

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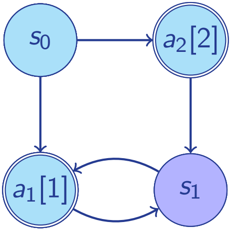

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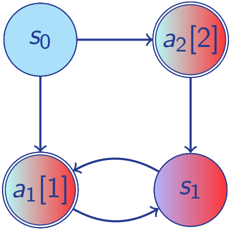
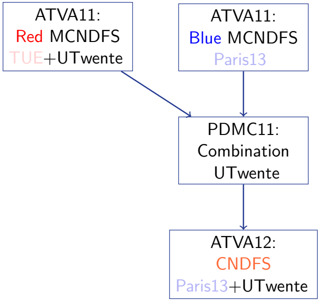
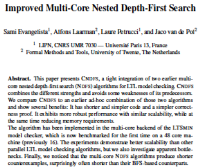


A History of Multi-core NDFSs





Improved Multi-Core Nested Depth-First Search

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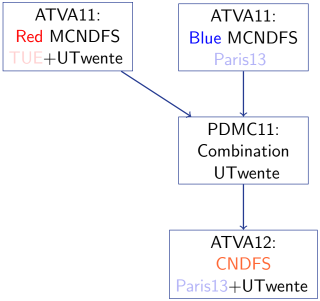

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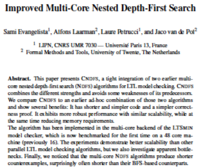

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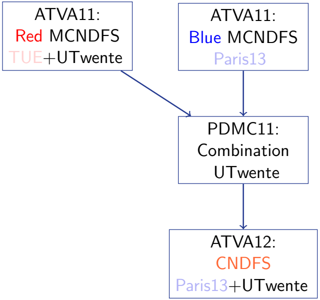

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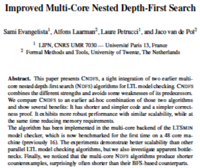

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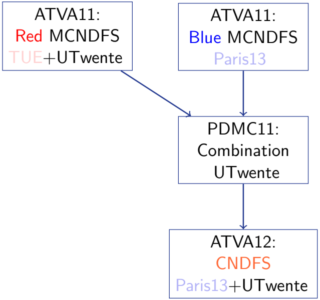

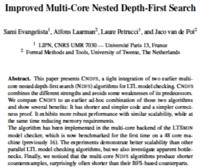

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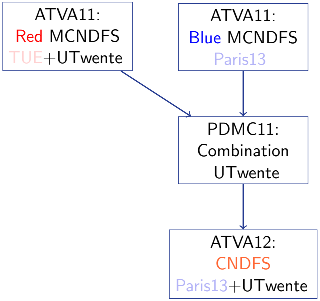

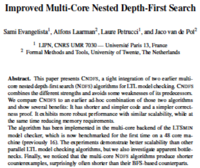

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- − Combined memory usage

CNDFS

- o Avoid repair in Blue MCNDFS by late Red coloring
- + Good, stable speedups
- + Memory usage: $P + 2 + \epsilon$ bits per state

CNDFS

procedure DFSblue(s,i)
    s.cyan[i] := true
    for all t $\in$ post(s) **do**
        if $\neg$t.cyan[i] $\wedge\neg$t.blue **then** DFSblue(t,i)
    if s $\in$ Acc **then**
        $R[i] := \emptyset$
        DFSred(s,i)
        await $\forall r \in R[i]: r \in$Acc$\wedge r \neq s \implies$ r.red
        forall $r \in R[i]$ **do** r.red := true
    s.blue := true
    s.cyan[i] := false
procedure DFSred(s,i)
    $R[i] := R[i] \cup \{s\}$
    for all t $\in$ post(s) **do**
        if t.cyan[i]=true **then** ExitCycle
        if t $\notin R[i]$ $\wedge$ $\neg$t.red **then** DFSred(t,i)

CNDFS

procedure DFSblue(s,i)
  s.cyan[i] := true
  for all t ∈ post(s) **do**
    if ¬t.cyan[i] ∧¬t.blue **then** DFSblue(t,i)   Loses post order!
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procedure DFSred(s,i) Collects states in $R[i]$
  $R[i] := R[i] \cup \{s\}$
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    if t.cyan[i]=true **then** ExitCycle
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CNDFS

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  if s ∈ Acc **then**
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    await $\forall r \in R[i]$: $r \in$Acc$\wedge r \neq s \implies$ r.red   Waiting repairs order!
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CNDFS Correctness Argument

Completeness & soundness:
$\exists \textbf{acc\_cycle}\colon s_0 \rightarrow^* \textbf{acc\_cycle} \Leftrightarrow \mathit{cndfs}(s_0, N) = \textbf{report}(\textbf{cycle})$

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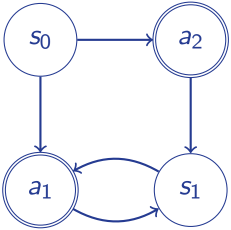
Termination?

CNDFS Example



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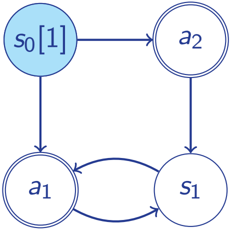

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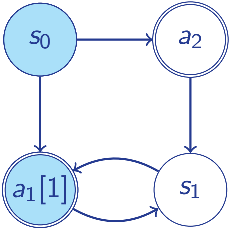

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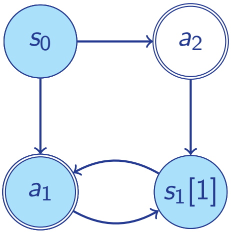

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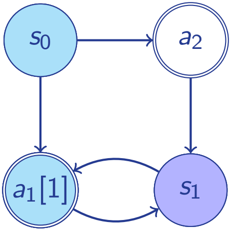

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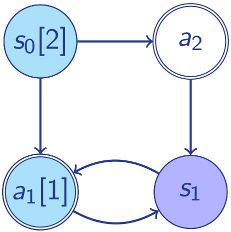

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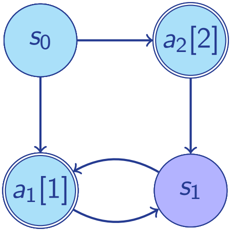

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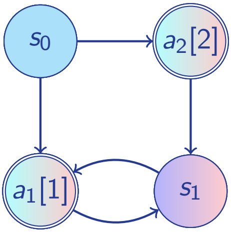

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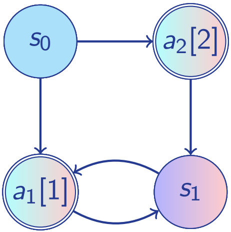
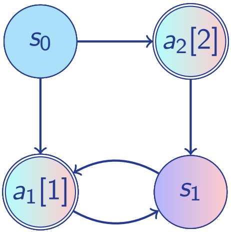


$a_1 \in R[2]$

CNDFS Example

$a_2 \in R[1]?$



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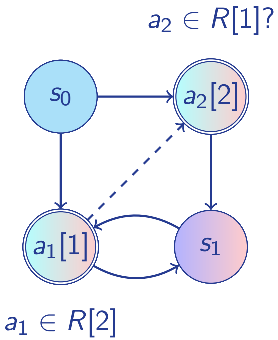
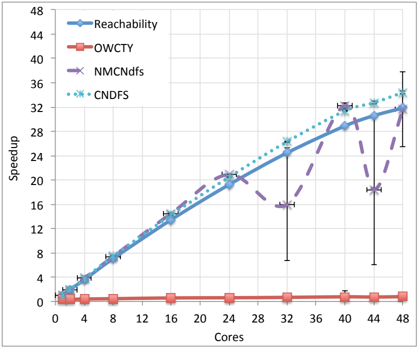


Experiments

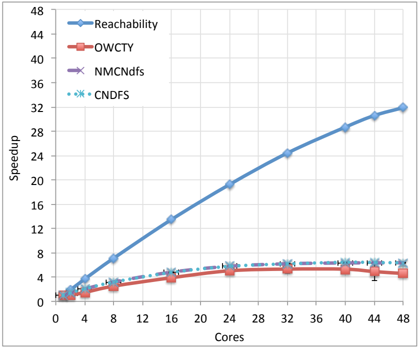
Experimental setup

- 48-core machine (NUMA)
- Compare: CNDFS and NMCNDFS in LTSmin, and OWCTY in DiVinE [Barnat et al.]
- 400+ models from the BEEM database
- Full verification and counter examples

Experiments: CNDFS vs OWCTY vs Reachability



leader_filters.6.prop2



lup.4.prop2

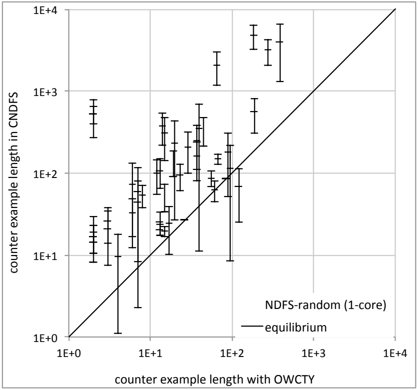
Full verification!

Experiments: CNDFS on-the-fly

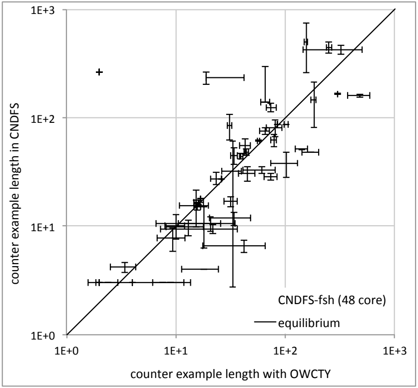
		NDFS	CNDFS		OWCTY	
		1 core	48 core		1 core	48 core
	model	Rand.	Rand.	Fsh.	Static	Rand.
Runtimes (sec)	anderson.8.prop3	36.4	4.1	0.2	2858.8	1433.2
	bakery.7.prop3	3.2	0.3	0.2	2.2	5.2
	bakery.8.prop4	15.7	0.6	0.3	73.4	14.3
	elevator2.3.prop3	8.4	1.4	0.2	432.3	192.5
	extinction.4.prop2	2.2	0.1	0.1	1.8	1.7
	peterson.6.prop4	29.1	0.9	0.5	668.4	705.7
	szymanski.5.prop4	1.7	1.3	0.2	2.1	376.4
Speedups	anderson.8.prop3		8.8	175.0		2.0
	bakery.7.prop3		10.9	21.2		0.4
	bakery.8.prop4		26.2	48.9		5.1
	elevator2.3.prop3		5.9	52.1		2.2
	extinction.4.prop2		18.5	28.8		1.0
	peterson.6.prop4		33.0	62.4		0.9
	szymanski.5.prop4		1.3	10.9		0.0

Super-linear speedups for bug hunting!

Experiments: Counterexample Length



Sequential



With 48x parallelism

Conclusions

- We have improved/investigated multi-core NDFS

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Future work

- Define class of scaling inputs
- Can we do better? Guaranteed speedup?

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Availability

- The implementation is available (open source) at:

  http://fmt.cs.utwente.nl/tools/ltsmin/

- See also: CAV'10, FMCAD'10, ATVA'11, NFM'11, SPIN'11, PDMC'11, PDMC'12